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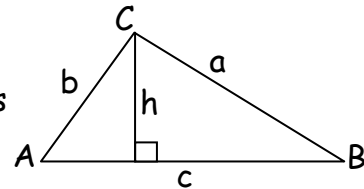
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Trigonometry: Right and Non-Right Triangles

Area of a Triangle Using Sine

We can use sine to determine the area of non-right triangles. This formula is derived from the area of a triangle formula, $A = \frac{1}{2}Bh$

For any triangle ABC with side a opposite $\angle A$, side b opposite $\angle B$ and side c opposite $\angle C$, height h is represented by a line perpendicular to the base of the triangle. If SAS is given and h is unknown,



$\sin \angle A$ can be written

$$\sin A = h/b$$

Multiplying produces

$$b \sin A = h$$

Substitute into the formula:

$$A = \frac{1}{2} c (b \sin A)$$

Rewritten:

$$A = \frac{1}{2} bc \sin A$$

Therefore,

$$A = \frac{1}{2} ab \sin (c)$$

$$A = \frac{1}{2} bc \sin (a)$$

$$A = \frac{1}{2} ac \sin (b)$$

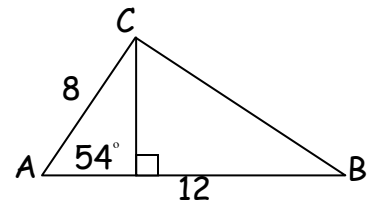
Note: You must know the included angle (the angle between the two known sides) in order to determine the area using this formula.

Example. Calculate the area of $\triangle ABC$

$$A = \frac{1}{2} bc \sin A$$

$$A = \frac{1}{2} (8)(12) \sin 54$$

$$A \approx 38.8$$



Law of Sines and Law of Cosines

When working with non-right triangles, we can use the Law of Sines and the Law of Cosines to determine unknown measurements:

Law of Sines

For any $\triangle ABC$ with side lengths a , b , and c ,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

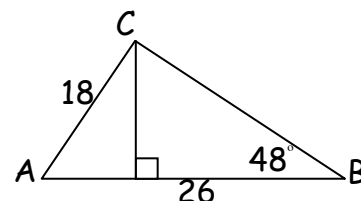
For any $\triangle ABC$ with side lengths a , b , and c :

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Example: Determine the area of $\triangle ABC$.



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Step 1: Determine $m\angle C$:

$$\frac{\sin 48}{18} = \frac{\sin C}{26}$$

$$\frac{26 (\sin 48)}{18} = \sin C$$

Find the inverse of $\sin C$:

$$\sin C = 0.5144$$

$$\sin^{-1} C = 30.96$$

Step 2: Determine $m\angle A$

$$180 - (48 + 30.96) = m\angle A$$

$$m\angle B \approx 101$$

Step 3: Find the area of $\triangle ABC$

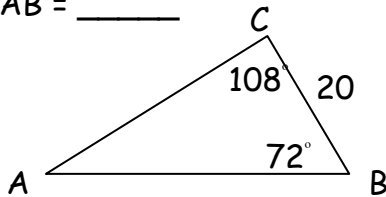
$$A = \frac{1}{2} bc \sin A$$

$$A = \frac{1}{2} (18)(26)(\sin 101)$$

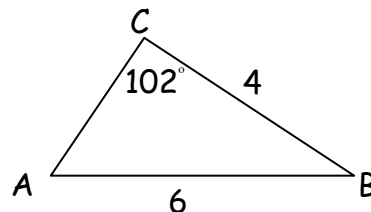
$$A \approx 229.7$$

Practice. Use the Law of Sines and the Law of Cosines to determine the missing measurements for $\triangle ABC$.

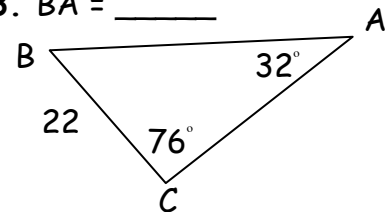
1. $AB =$ _____



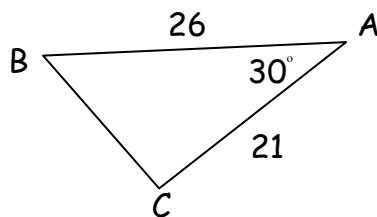
2. $m\angle A =$ _____



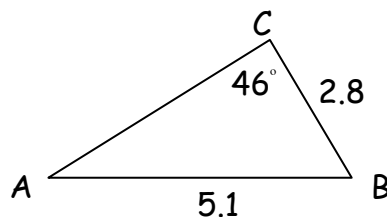
3. $BA =$ _____



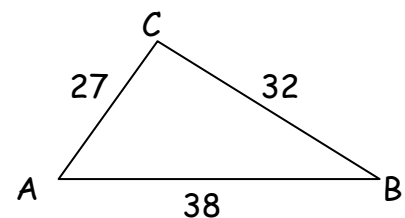
4. $BC =$ _____



5. $m\angle A =$ _____

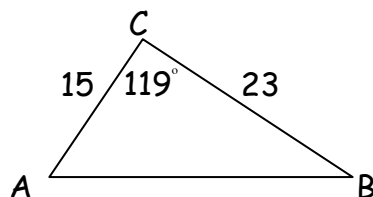


6. $m\angle A =$ _____

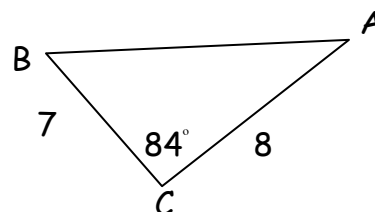


7-9. Identify the area of the following triangles.

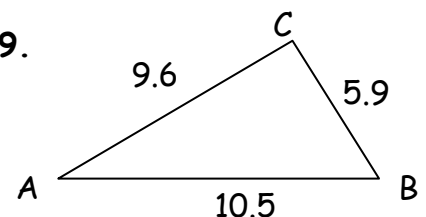
7.



8.



9.



10. Using the same reasoning given above, derive the formula for the area of triangle ABC given measurements b , $m\angle A$, and c .

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Answer Key

Trigonometry: Right and Non-Right Triangles

1. 32.4

2. 40.7°

3. 40.3

4. 13.1

5. 23.3°

6. 79.7°

7. 150.9

8. 27.8

9. 33.8

10. a. $A = \frac{1}{2} Bh$; B = side b; $h = ?$

b. $\sin A = \frac{h}{c}$

c. $c(\sin A) = c(\frac{h}{c})$

d. $h = c(\sin A)$

e. $A = \frac{1}{2} (b)(c)(\sin A)$