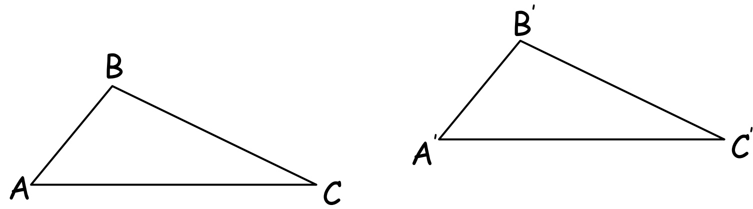
Name:______________________ Date:________________

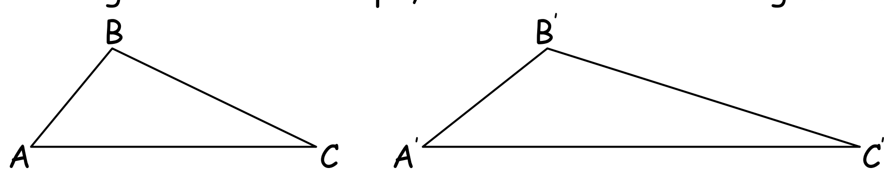
Congruence: Rigid Motions

Rigid motion refers to the transformation of an object so that its size and shape are not changed. For example:

A. This triangle is translated to the right and up. Its size and shape, however, are not changed. Therefore, we say that this has rigid motion.



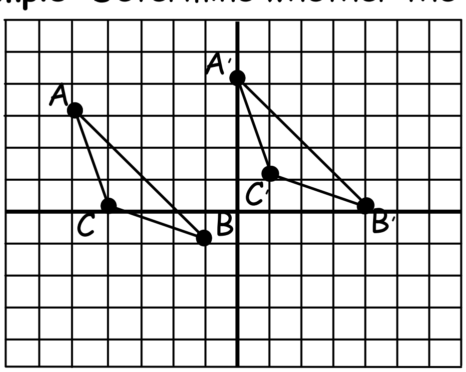
B. This triangle is translated to the right. It is then stretched. Because it does not retain its original size and shape, it is considered non-rigid.



Notice that the side lengths and the angle measures in Example A above do not change. They are congruent. *Transformations are rigid if and only if the preimage and image are congruent.*

When inputting transformations on a coordinate plane, we can predict whether a transformation will be rigid. If it is rigid triangle, two corresponding side lengths and two corresponding angles are congruent.

Example: Determine whether the translation is rigid.



Step 1: Map it using Coordinate Notation

$A\ (-5,3) \rightarrow (x+5, y+2) \rightarrow A'\ (0, 5)$

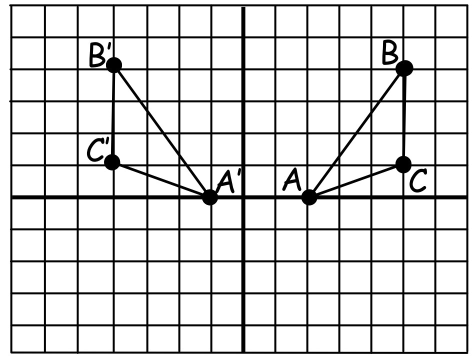
$B\ (-1, -1) \rightarrow (x+5, y+2) \rightarrow B'\ (4, 1)$

$C\ (-4, 0) \rightarrow (x+5, y+2) \rightarrow C'\ (1, 2)$

Because the translation has consistent mapping, this is a rigid motion.

Name:_____________________ Date:________________

Example: Determine whether the reflection is rigid.



Step 1: Notice that ΔABC has retained its shape, despite being reflected along $x= \frac{1}{2}$.

Step 2: Verify two leg lengths.

$BC = 3$ and $B'C' = 3$

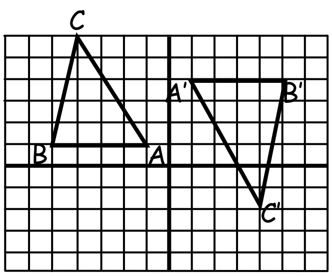
$AC = \sqrt{(5-2)^2 + (1-0)^2} = \sqrt{10}$

d $A'C' = \sqrt{(-4+1)^2 + (0-1)^2} = \sqrt{10}$

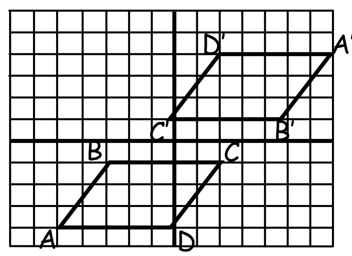
By SSS, we can verify that ΔABC and $\Delta A'B'C'$ are congruent. Therefore, it has rigid motion.

Practice. Determine whether the following transformations have rigid or non-rigid motion. Defend each answer mathematically.

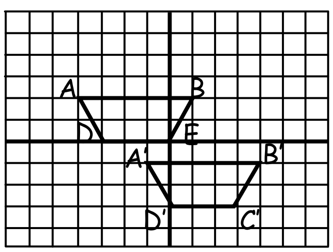
1.



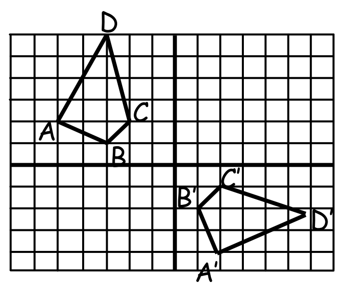
2.



3.



4.



Given the transformation statement, predict the location of the image.

5. $\Delta ABC \rightarrow (x-3, y+2)$

A (0,3), B(6,4), C(-2, 8)

6. LMNO reflected across y=-2

L(-2, 4), M(-1,1), N(6,2), O (-3, 8)

7. ΔTUV reflected across x=1

T(7,9), U(1,-3), V(3, 5)

8. TRIP $\rightarrow (x+4, y-5)$

T(0,-5), R(-4, 6), I(-2, 10), P(5, -2)

9. STEW rotated 180° about the origin

S(-1, -3), T(-2,4), E(0, 5), W(-2, 0)

10. ΔCRW rotated left 90° about the origin

C(1, 1), R(4, -3), W(-5, -4)

Name:____________________ Date:_______________

Answer Key

Congruence: Rigid Motions

1. Yes. SSS. AB ≅ A'B'; AC ≅ A'C'; BC ≅ B'C'
2. Yes. AB ≅ A'B'; BC ≅ B'C'; CD ≅ C'D'; DA ≅ D'A'
3. No. The points are not labeled correctly.
4. Yes. AB ≅ A'B'; BC ≅ B'C'; CD ≅ C'D'; DA ≅ D'A'
5. A'(-3,5), B'(3,6), C'(-5,10)
6. L'(-2,-8), M'(-1,-5), N'(6,-6), O'(-3,-12)
7. T'(-5,9), U'(1,-3), V'(-1,5)
8. T'(4,-10), R'(0, 1), I'(2, 5), P'(9,-7)
9. S'(3,1), T'(-4,2), E'(-5,0), W'(0,2)
10. C'(-1,1); R'(3,4); W'(4,-5)