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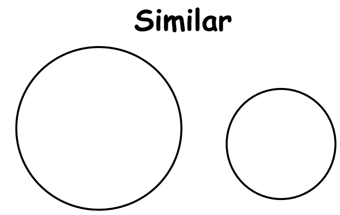
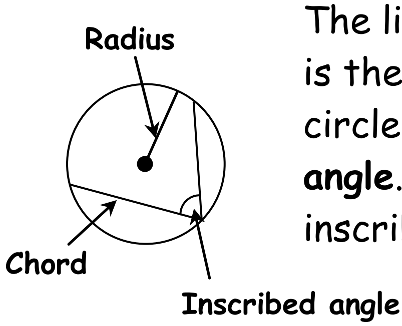
Circles: Theorems about Circles

To be **similar**, two objects do not need to have the same size, but must have the same shape. In order for something to be a **circle**, it must have a center that is equidistant to any point on its circumference. Therefore, all circles are similar.

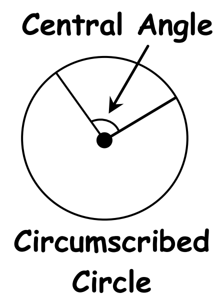


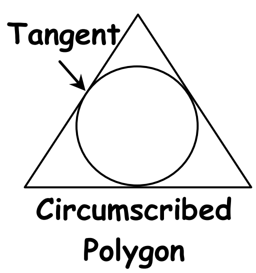


The line connecting the center to the circumference of the circle is the **radius**. A **chord** is a segment with endpoints that lie on the circle. Combining two chords within a circle creates an **inscribed angle**. The vertex of an inscribed angle rests on the circle. An inscribed angle that rests on the diameter is a right angle.

Combining chords into a polygon creates a **circumscribed circle**.

Combining two radii creates a **central angle**. The vertex of the central angle rests on the center of the circle.





A **tangent** is a line that is in the same plane as a circle and intersects the circle at exactly one point. The tangent of a circle is always perpendicular to the radius. In a **circumscribed polygon,** the sides of the polygon are made up of the tangents of a circle.

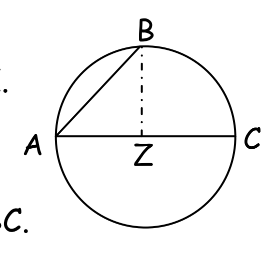
Challenge: Given that $\angle ABC$ is inscribed in circle Z, prove that $m\angle ABC$ is half the measure of $\overset{\frown}{AC}$.

Step 1: Draw BZ. $m\overset{\frown}{AC} = m\angle AZC$;

Step 2: Use Exterior Angle Theorem $m\angle AXC = m\angle ABZ + m\angle BAZ$.

Step 3: Since $\overline{ZA}$ and $\overline{ZB}$ are radii, $\overline{ZA} \cong \overline{ZB}$, then ΔAZB is isosceles.
Thus, $m\angle ABZ = m\angle BAZ$

Step 4: Substitution $m\overset{\frown}{AC} = 2m\angle ABZ$ or $2m\angle ABC$.
Thus, $\frac{1}{2}\, m\overset{\frown}{AC} = m\angle ABC$.



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Practice. Complete the proof.

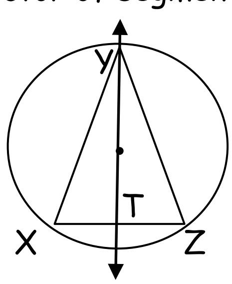
Tangent-Radius Theorem: The radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Given: Segment AB intersects circle X at point C.
Prove: m∠XCA = m∠XCB

1. Segment AB intersects circle X at point C	1. ____________________________
2. ____ is the radius of circle X	2. Def of a radius
3. ∠XCA is a right angle	3. ____________________________
4. m∠XCA = 90°	4. ____________________________
5. ______________________	5. Def of Supp ∠s
6. m∠AB - m∠XCA = 90°	6. ____________________________
7. ______________________	7. Simplify
8. m∠XCB = m∠XCB	8. ____________________________

A circle that contains all the vertices of a polygon is circumscribed about the polygon. The circumcenter of XYZ is the center of the circumscribed circle.

9-10. Complete the proof using the figure below:
Given: YT bisects ∠XYZ; XY ≅ YZ
Prove: Line YT is the bisector of segment XZ



YT bisects ∠XYZ; XY ≅ YZ	Given
∠XYT ≅ ∠ZYT	9.______________
XY ≅ ZY	Reflexive POC
ΔXYT ≅ ΔZYT	SAS
∠XTY ≅ ∠ZTY	CPCTC
∠XTY and ∠ZTY are supp.	Lin. Pair Thm.
∠XTY and ∠ZTY are rt. ∠s	≅ ∠s supp → rt ∠s
m∠XTY = 90; m∠ZTY = 90	Def of rt. ∠s
YT ⊥ XZ	Definition of ⊥
XT ≅ TZ	CPCTC
T is the midpoint of XZ	Def of midpoint
10. _______________	Def of ⊥ bisector

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Answer Key

Circles: Theorems about Circles

1. Given
2. XC
3. Tangent-radius Thm
4. Def of Rt ∠
5. m∠AB = 180^{o}
6. Angle Subtraction
7. m∠XCB = 90^{o}
8. Substitution
9. Def of angle bisector
10. Line YT is the bisector of segment XZ